NAG Toolbox for MATLAB

f07pj

1 Purpose

f07pj computes the inverse of a real symmetric indefinite matrix A, where A has been factorized by f07pd, using packed storage.

2 Syntax

```
[ap, info] = f07pj(uplo, ap, ipiv, 'n', n)
```

3 Description

f07pj is used to compute the inverse of a real symmetric indefinite matrix A, the function must be preceded by a call to f07pd, which computes the Bunch–Kaufman factorization of A, using packed storage.

If **uplo** = 'U', $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $U^{\mathrm{T}}P^{\mathrm{T}}XPU = D^{-1}$.

If **uplo** = 'L', $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $L^{\mathrm{T}}P^{\mathrm{T}}XPL = D^{-1}$.

4 References

Du Croz J J and Higham N J 1992 Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how A has been factorized.

uplo = 'U'

$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

uplo = 'L'

$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **ap**$(*)$ **– double array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The factorization of A stored in packed form, as returned by f07pd.

3: **ipiv**$(*)$ **– int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Details of the interchanges and the block structure of D, as returned by f07pd.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **ipiv**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

work

5.4 Output Parameters

1: **ap**$(*)$ **– double array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The factorization contains the n by n matrix A^{-1}.

More precisely,

if **uplo** = 'U', the upper triangle of {\it A}$^{-1}$ must be stored with element A_{ij} in **ap**$(i+j(j-1)/2)$ for $i \leq j$;
if **uplo** = 'L', the lower triangle of {\it A}$^{-1}$ must be stored with element A_{ij} in **ap**$(i+(2n-j)(j-1)/2)$ for $i \geq j$.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **ipiv**, 5: **work**, 6: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, $d(i,i)$ is exactly zero; D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if **uplo** = 'U', $\left|DU^{\mathrm{T}}P^{\mathrm{T}}XPU - I\right| \leq c(n)\epsilon\left(|D|\left|U^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|U| + |D|\left|D^{-1}\right|\right)$;
if **uplo** = 'L', $\left|DL^{\mathrm{T}}P^{\mathrm{T}}XPL - I\right| \leq c(n)\epsilon\left(|D|\left|L^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|L| + |D|\left|D^{-1}\right|\right)$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogues of this function are f07pw for Hermitian matrices and f07qw for symmetric matrices.

9 Example

```
uplo = 'L';
ap = [2.07;
     4.2;
     0.2230413840558341;
     0.6536583767489105;
     1.15;
     0.8115010321439103;
     -0.5959697237786296;
     -2.59067708640519;
     0.3030846795506181;
     0.4073851981348882];
ipiv = [int32(-3);
     int32(-3);
     int32(3);
     int32(4)];
[apOut, info] = f07pj(uplo, ap, ipiv)
```

```
apOut =
    0.7485
    0.5221
   -1.0058
   -1.4386
   -0.1605
   -0.3131
   -0.7440
    1.3501
    2.0667
    2.4547
info =
           0
```